

# DIGITAL IMAGES: QUANTITATIVE/QUALITATIVE MEASURES

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## TODAY'S SCHEDULE

### QUANTITATIVE/QUALITATIVE MEASUREMENTS

Cumulative Energy

Peak Signal to Noise Ratio (PSNR)

Cumulative Energy and PSNR Computer Lab

## TODAY'S SCHEDULE

9:00-10:15 ⇒ **Lecture One:** Cumulative Energy, Quantization, and Peak Signal-to-Noise Ratio

10:15-10:30 Coffee Break (SCA 202)

10:30-11:45 **Lecture Two:** Huffman Coding

12:00-1:00 Lunch

1:30-2:45 **Lecture Three:** Putting it All Together: Image Compression

2:45-3:00 Coffee Break (SCA 202)

3:00-4:15 **Lecture Four:** Daubechies Wavelet Transformations



# QUANTITATIVE/QUALITATIVE MEASUREMENTS

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  - ▶ Note  $\|\mathbf{v}\| = \|\mathbf{y}\|$ .
  - ▶ Define

$$CE_k = \sum_{j=1}^k \frac{y_j^2}{\|\mathbf{y}\|^2}$$

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## CUMULATIVE ENERGY

$$\begin{aligned}CE_1 &= \frac{y_1^2}{y_1^2 + y_2^2 + \cdots + y_N^2} \\CE_2 &= \frac{y_1^2 + y_2^2}{y_1^2 + y_2^2 + \cdots + y_N^2} \\&\vdots \\CE_N &= \frac{y_1^2 + y_2^2 + \cdots + y_N^2}{y_1^2 + y_2^2 + \cdots + y_N^2} = 1\end{aligned}$$

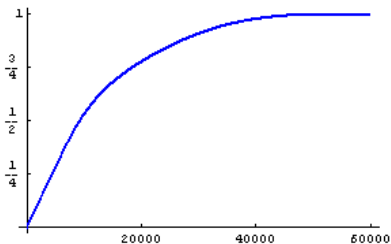
# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## CUMULATIVE ENERGY

Below is an  $200 \times 300$  image of the *Four Canoes* and its cumulative energy vector.



Image



Cumulative Energy

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

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How we typically use the Cumulative Energy vector is by

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- ▶ Identifying those elements in the vector that produce that energy
- ▶ Converting the remaining elements to 0 (quantization).

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## CUMULATIVE ENERGY

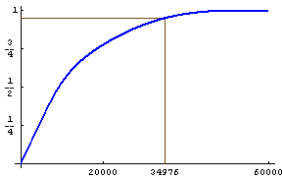
Here is a really naive example. We keep 95% of the energy of the image. That's the largest 34976 (in absolute value) elements of image A. We convert the rest to 0 and plotted the results.

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## CUMULATIVE ENERGY



Image



Cumulative Energy



Naive Quantization

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## CUMULATIVE ENERGY

Why would someone want to change the image in this way?

- ▶ Typically, we transform the image first and then apply a threshold (quantizer).

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## CUMULATIVE ENERGY

Why would someone want to change the image in this way?

- ▶ Typically, we transform the image first and then apply a threshold (quantizer).
- ▶ As we will soon see, compression algorithms really exploit images that consist of large numbers of relatively few values.

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## PEAK SIGNAL TO NOISE RATIO (PSNR)

- ▶ The Peak Signal to Noise Ratio (PSNR) is a qualitative measure that tells us how well a compression scheme has done its job.

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## PEAK SIGNAL TO NOISE RATIO (PSNR)

- ▶ The Peak Signal to Noise Ratio (PSNR) is a qualitative measure that tells us how well a compression scheme has done its job.
- ▶ Suppose  $A$  is an  $N \times M$  image and we have employed a compression scheme to obtain  $N \times M$  image  $B$ .

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## PEAK SIGNAL TO NOISE RATIO (PSNR)

- ▶ We first define the error function

$$Err(A, B) = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (A_{ij} - B_{ij})^2$$

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- ▶ The Peak Signal to Noise Ratio is defined to be

$$PSNR = 10 \log_{10} \left( \frac{255^2}{Err(A, B)} \right)$$

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## PEAK SIGNAL TO NOISE RATIO (PSNR)

Clearly, the smaller the error function, the higher the PSNR, so when we compare two compression schemes, the one with the higher PSNR is better.

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## PEAK SIGNAL TO NOISE RATIO (PSNR)

Here is an example (using CE with energy 95% and 90%).



Image



(95%) PSNR = 18.46



(90%) PSNR = 15.42

# QUANTITATIVE/QUALITATIVE MEASUREMENTS

## CUMULATIVE ENERGY AND PSNR COMPUTER LAB

Please go to Matlab or Mathematica and open the corresponding lab on Cumulative Energy and PSNR.