DAUBECHIES FILTERS

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In Session 1, we constructed the orthogonal Haar Wavelet Transform:

$$W_8 = \left[\begin{array}{cccccccc} h_1 & h_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_1 & h_0 \\ \hline g_1 & g_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_1 & g_0 \end{array}\right]$$

where

$$\mathbf{h} = (h_0, h_1) = (\sqrt{2}/2, \sqrt{2}/2), \qquad \mathbf{g} = (g_0, g_1) = (\sqrt{2}/2, -\sqrt{2}/2)$$

- $\mathbf{h} = (h_0, h_1) = (\sqrt{2}/2, \sqrt{2}/2)$ is called a lowpass filter.
- Lowpass filters tend to preserve non-oscillatory data and either dampen or annihilate oscillatory data.
- Lowpass filters are best analyzed using a Fourier series.

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- Lowpass filters are best analyzed using a Fourier series.

- Pretending we're engineers for the moment, let's "form" the Fourier series

$$H(\omega) = \sum_{k=0}^{1} h_k e^{ik\omega} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} e^{i\omega}$$

- We wish to plot $|H(\omega)|$ so simplifying gives

$$\begin{aligned} H(\omega) &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} e^{i\omega} \\ &= \frac{\sqrt{2}}{2} e^{i\omega/2} \left(e^{-i\omega/2} + e^{i\omega/2} \right) \\ &= \sqrt{2} e^{i\omega/2} \cos(\omega/2) \end{aligned}$$

- So

$$\boxed{|H(\omega)| = \sqrt{2} \cos(\omega/2)}$$

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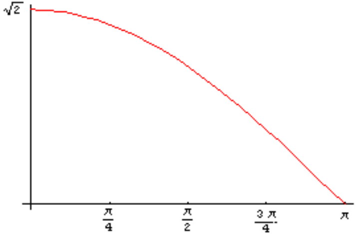

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- Suppose $\mathbf{h} = (h_n, \ldots, h_m)$ with Fourier series

$$H(\omega) = \sum_{k=n}^{m} h_k \, e^{ik\omega}$$

- We will say $\mathbf{h}$ is a lowpass filter if

$$\boxed{H(0) = \sum_{k=n}^{m} h_k = \sqrt{2}}$$

and

$$\boxed{H(\pi) = \sum_{k=n}^{m} h_k e^{ik\pi} = \sum_{k=n}^{m} h_k(-1)^k = 0}$$

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- If we write down the Fourier series for $\mathbf{g} = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, we have

$$G(\omega) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} e^{-i\omega}$$

- Simplifying gives

$$\begin{aligned} G(\omega) &= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} e^{-i\omega} \\ &= \frac{\sqrt{2}}{2} e^{-i\omega/2} \left(e^{i\omega/2} - e^{-i\omega/2} \right) \\ &= \sqrt{2} i e^{-i\omega/2} \sin(\omega/2) \end{aligned}$$

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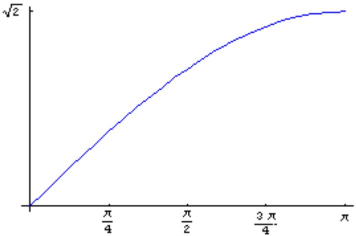

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- We can build longer orthogonal filters **h**, **g**.
- Orthogonal filters are those that give rise to an orthogonal wavelet transformation W_N (N even).
- It turns out length 3 doesn't work, so we will try length 4.
- Let lowpass $\mathbf{h} = (h_0, h_1, h_2, h_3)$ and highpass $\mathbf{g} = (g_0, g_1, g_2, g_3)$.

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We form the matrix

$$W_8 = \left[\begin{array}{cccccccc} h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \\ \hline g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_3 & g_2 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_3 & g_2 & g_1 & g_0 \\ g_1 & g_0 & 0 & 0 & 0 & 0 & g_3 & g_2 \end{array}\right]$$

- The order of the filters is not important - it turns out reflections work as well.
- The reverse order I've used reflects the fact that the wavelet matrix was first motivated using convolution (see Section 5.1 of the text).
- Note the "wrapping of rows". This makes it easy to set up orthogonality conditions for W_8, but is not practical in applications.
- In this case, the fourth and eighth elements of the transformed data will be built using elements from the beginning and end of the input vector.
- This makes sense if the data are periodic ...
- But data are not typically periodic!
- We can periodize input data, but then we need more from our filters ... that's coming!

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What orthogonality conditions can we impose on W_8?

$$W_8 = \left[\begin{array}{cccccccc} h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \\ \hline g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_3 & g_2 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_3 & g_2 & g_1 & g_0 \\ g_1 & g_0 & 0 & 0 & 0 & 0 & g_3 & g_2 \end{array}\right]$$

Orthogonality Conditions

$$\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 &= 1 \\ h_0 h_2 + h_1 h_3 &= 0 \end{aligned}$$

$$\begin{aligned} g_0^2 + g_1^2 + g_2^2 + g_3^2 &= 1 \\ g_0 g_2 + g_1 g_3 &= 0 \end{aligned}$$

$$\begin{aligned} h_0 g_0 + h_1 g_1 + h_2 g_2 + h_3 g_3 &= 0 \\ h_0 g_2 + h_1 g_3 &= 0 \\ h_2 g_0 + h_3 g_1 &= 0 \end{aligned}$$

and Lowpass/Highpass Conditions

$$\begin{aligned} h_0 + h_1 + h_2 + h_3 &= \sqrt{2} \\ h_0 - h_1 + h_2 - h_3 &= 0 \end{aligned}$$

$$\begin{aligned} g_0 + g_1 + g_2 + g_3 &= 0 \\ g_0 - g_1 + g_2 - g_3 &= \sqrt{2} \end{aligned}$$

Look at the 5th equation:

$$\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 &= 1 \\ h_0 h_2 + h_1 h_3 &= 0 \end{aligned}$$

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Given h_0, h_1, h_2, h_3, can you find g_0, g_1, g_2, g_3 to satisfy it?

- How about

$$g_0 = h_3, g_1 = -h_2, g_2 = h_1, g_3 = -h_0$$

- Then

$$h_0 g_0 + h_1 g_1 + h_2 g_2 + h_3 g_3 = h_0(h_3) + h_1(-h_2) + h_2(h_1) + h_3(-h_0) = 0$$

- Moreover, if $h_0 + h_1 + h_2 + h_3 = \sqrt{2}$ and $h_0 - h_1 + h_2 - h_3 = 0$, then

$$g_0 + g_1 + g_2 + g_3 = h_3 - h_2 + h_1 - h_0 = 0$$

and

$$g_0 - g_1 + g_2 - g_3 = h_3 - (-h_2) + h_1 - (-h_0) = \sqrt{2}$$

- **g** is highpass!

- How about

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- There's more good news ...
- The orthogonality conditions reduce to

$$\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 &= 1 \\ h_0 h_2 + h_1 h_3 &= 0 \end{aligned}$$

- Add to that the lowpass conditions:

$$\begin{aligned} h_0 + h_1 + h_2 + h_3 &= \sqrt{2} \\ h_0 - h_1 + h_2 - h_3 &= 0 \end{aligned}$$

  and we have a (quadratic) system to solve.
- The moral of the story is we can build **g** from **h**.

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- The good news (for engineers) is that the system has an infinite number of solutions.
- We can add a condition!
- The condition Ingrid Daubechies added:
- "Flatten" $H(\omega)$ at $\omega = \pi$.
- Set $H'(\pi) = 0$!

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- Differentiating

$$H(\omega) = h_0 + h_1\, e^{i\omega} + h_2\, e^{2i\omega} + h_3\, e^{3i\omega}$$

  gives

$$H'(\omega) = ih_1\, e^{i\omega} + 2ih_{,} e^{2i\omega} + 3ih_3\, e^{3i\omega}$$

- Plugging in $\omega = \pi$ and simplifying gives the condition

$$h_1 - 2h_2 + 3h_3 = 0$$

- Differentiating

$$H(\omega) = h_0 + h_1\, e^{i\omega} + h_2\, e^{2i\omega} + h_3\, e^{3i\omega}$$

gives

$$H'(\omega) = ih_1\, e^{i\omega} + 2ih_{,}e^{2i\omega} + 3ih_3\, e^{3i\omega}$$

- Plugging in $\omega = \pi$ and simplifying gives the condition

$$h_1 - 2h_2 + 3h_3 = 0$$

So we want to solve

$$\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 &= 1 \\ h_0 h_2 + h_1 h_3 &= 0 \\ h_0 + h_1 + h_2 + h_3 &= \sqrt{2} \\ h_0 - h_1 + h_2 - h_3 &= 0 \\ h_1 - 2h_2 + 3h_3 &= 0 \end{aligned}$$

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Let's look at the notebook

`DaubechiesFilters.nb`