

## Math 316-01 Homework Set 6

**Instructions:** Work each of the assigned problems. Write the solution to each problem in an organized and concise manner. Print out and attach (output deleted) any Mathematica code you used to solve the problems. Homework sets that are messy and disorganized will not be graded. This homework set is due on **Friday, May 1, 2009** at 5:00pm.

1. 10.10

2. 10.16

3. Let  $\tilde{\mathbf{h}} = (\tilde{h}_0, \tilde{h}_1) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$  and suppose  $\mathbf{h}$  is a symmetric filter of length 10. Write down all such filters  $\mathbf{h}$  that satisfy (10.10). *Hint:* Example 10.4 will be helpful.

4. Consider

$$\tilde{\mathbf{h}} = (\tilde{h}_{-1}, \tilde{h}_0, \tilde{h}_1, \tilde{h}_2) = \left(\frac{\sqrt{2}}{8}, \frac{3\sqrt{2}}{8}, \frac{3\sqrt{2}}{8}, \frac{\sqrt{2}}{8}\right)$$

(a) Verify  $\tilde{\mathbf{h}}$  satisfies  $H(0) = \sqrt{2}$  and  $H(\pi) = 0$  and is thus a lowpass filter.

(b) Let

$$\mathbf{h} = (h_{-3}, h_{-2}, h_{-1}, h_0, h_1, h_2, h_3, h_4) = (h_4, h_3, h_2, h_1, h_1, h_2, h_3, h_4)$$

Write down the system of linear equations obeyed by  $\mathbf{h}$  if the Fourier series  $\tilde{H}(\omega)$ ,  $H(\omega)$  associated with  $\tilde{\mathbf{h}}$ ,  $\mathbf{h}$ , respectively, satisfy (10.10). *Hint:* Your system should consist of three linear equations.

(c) Solve this system and verify that there are an infinite amount of solutions.

(d) We know from Problem 10.10 that  $H(\pi) = 0$ . Add the equation  $H'(\pi) = 0$  to the system in part (b) and solve this system. How many answers are there?

(e) Write down the wavelet filters  $\tilde{\mathbf{g}}$  and  $\mathbf{g}$ .

(f) Write down the matrices  $\tilde{W}_8$  and  $W_8$ .

(g) Use Mathematica to verify that  $\tilde{W}_8^{-1} = W_8^T$ .