Math 210-02 Quiz #7 KEY **Name:**______________________________

Friday, February 19, 2010

1. Solve the initial value problem $y' = (1-y)\cos x$, $y(\pi) = 2$.

   We rewrite the equation as $y' = \cos x - y\cos x$ or

   $$y' + \cos x y = \cos x$$

   So $P(x) = \cos x$ so that $\rho(x) = e^{\int P} = e^{\int \cos x} = e^{\sin x}$. We multiply the above equation by $\rho(x)$ to obtain

   $$\begin{aligned} e^{\sin x}y' + e^{\sin x}y &= e^{\sin x}\cos x \\ \left(e^{\sin x}y\right)' &= e^{\sin x}\cos x \\ e^{\sin x}y &= \int e^{\sin x}\cos x \end{aligned}$$

   The integral on the right is a basic u-substitution with $u = \sin x$. We have

   $$\begin{aligned} e^{\sin x}y &= e^{\sin x} + C \\ y &= e^{-\sin x}\left(e^{\sin x} + C\right) \\ y &= 1 + Ce^{-\sin x} \end{aligned}$$

   Substituting the initial condition gives $2 = 1 + Ce^{-\sin\pi} = 1 + C$ so that $C = 1$. Thus the solution to the IVP is

   $$y = 1 + e^{-\sin x}$$