Math 210-02 Quiz #2 KEY **Name:**____________________________

Monday, February 8, 2010

Solve each of the following problems. Show all work for full credit!

1. Find a function $y = f(x)$ that satisfies the initial value problem $\frac{dy}{dx} = x\sqrt{x^2+9}$, $y(-4) = 0$.

   We get the general solution via integration with a $u = x^2 + 9$ substitution:

   $$y(x) = \int x\sqrt{x^2+9}\,dx = \frac{1}{2}\cdot\frac{2}{3}(x^2+9)^{3/2} + C = \frac{1}{3}(x^2+9)^{3/2} + C$$

   Since $y(-4) = 0$, we have

   $$0 = \frac{1}{3}((-4)^2+9)^{3/2} + C = \frac{1}{3}\cdot 125 + C$$

   so that $C = -\frac{125}{3}$. The particular solution is $y(x) = \frac{1}{3}(x^2+9)^{3/2} - \frac{125}{3}$.

2. On planet Gzyx, a ball dropped from a height of 20 feet hits the ground in 2 seconds. If a ball dropped from the top of a 200–ft–tall building on Gzyx, how long will it take to hit the ground?

   We have $v(t) = -at$ since the initial velocity is zero and acceleration is constant. Integrating again and using the fact that the initial position is 20–ft, we have $p(t) = -\frac{a}{2}t^2 + 20$. After $t = 2$ seconds, the position is zero, so we can use this information to solve for acceleration:

   $$0 = -\frac{a}{2}\cdot 2^2 + 20 = -2a^2 + 20$$

   so that $a = 10$. From the 200–ft building, the position function is

   $$p(t) = -5t^2 + 200$$

   Setting this equation to zero and solving gives $t = \sqrt{40} \approx 6.323$ seconds.