

1. Transform  $x'' + 3x' + 4x - 2y = 0$ ,  $y'' + 2y' - 3x + y = \sin t$  into a system of first-order differential equations.

Let  $x_1 = x$ ,  $x_2 = x' = x'_1$  and  $y_1 = y$ ,  $y_2 = y' = y'_1$ . Then

$$\begin{aligned}x'_1 &= x_2 \\x'_2 &= -3x_2 - 4x_1 + 2y_1 \\y'_1 &= y_2 \\y'_2 &= -2y_1 + 3x_1 - y_1 + \sin t\end{aligned}$$

2. Solve  $x' = y$ ,  $y' = 6x - y$ ,  $x(0) = 1$ ,  $y(0) = 2$ .

Differentiating the first equation gives  $x'' = y' = 6x - y = 6x - x'$ , so

$$x'' + x' - 6x = 0$$

The characteristic equation is  $r^2 + r - 6 = (r + 3)(r - 2) = 0$  so the roots are  $r = 2, -3$  and we have

$$x(t) = c_1 e^{2t} + c_2 e^{-3t}$$

Differentiating this equation gives

$$y(t) = x'(t) = 2c_1 e^{2t} - 3c_2 e^{-3t}$$

Plugging zero into both equations and setting  $x(0) = 1$ ,  $y(0) = 2$ , gives

$$\begin{aligned}c_1 + c_2 &= 1 \\2c_1 - 3c_2 &= 2\end{aligned}$$

Multiplying the top equation by 3 and adding it to the second equation yields  $5c_1 = 5$  or  $c_1 = 1$ . In this case,  $c_2 = 0$  so the solution to the system is

$$x(t) = e^{2t} \quad \text{and} \quad y(t) = 2e^{2t}$$