Multiridgelets and Image Processing - Patrick Van Fleet

Background

The discrete biorthogonal wavelet transform in applications such as the JPEG2000 Image Compression Standard and the FBI Fingerprint Image Standard. In both cases, the developers chose biorthogonal wavelet transforms for the following reasons:

1. Fast algorithms exist for implementing discrete biorthogonal transforms on the computer.

2. The filter pairs used to construct the transform are symmetric/anti-symmetric. This property can be exploited when construction compression algorithms that are sensitive to boundary conditions.

3. Biorthogonal wavelet transforms (and orthogonal wavelet transforms) classify data in various subbands. One subband is an approximation to the original image. The remaining subbands hold detail information at various levels. These details can be combined with the approximation subband to reconstruct the original image. By construction, most of the energy is contained in the approximation subband and the remaining subbands are relatively sparse. Thus smart quantization in these subbands has little effect on the resolution of the reconstructed image.

4. The transform filter pair come from a so-called *scaling function* and *wavelet function*. These functions are compactly and minimally supported and they can be constructed to satisfy certain regularity conditions.

5. The subband structure of the transform allows for progressive transmission of the image. That is, we can perform only a partial reconstruction if we do not require full resolution.

6. The biorthogonal wavelet transform is quite local in nature. For that reason, we can reconstruct a so-called *region of interest* without having to reconstruct the entire image.

Daubechies [16]) proved that it was impossible to construct suitably regular scaling and wavelet functions that are orthogonal, symmetric/antisymmetric, and enjoy compact support. Only three of the properties can be satisfied at any one time. Biorthogonal transforms are an answer to this problem - here orthogonality is surrendered in order to construct symmetric/antisymmetric filters.

As a result of these shortcomings, research efforts in the mid-1990's focused on the development of so-called *multiwavelets*. In this approach, A scaling functions and wavelet functions were constructed. It was shown in [19] that in the multiwavelet setting, suitably regular scaling functions and wavelet functions could be constructed that were orthogonal, symmetric/antisymmetric, and compactly supported.

Another disadvantage of both wavelets and multiwavelets is that they are severely limited in orientation. The detail subbands for both types of transforms are oriented horizontally, vertically, and diagonally (along say $y = x$). Candès [11], Donoho [18], and Donoho and Candès [12] proposed a number of variations to the wavelet concept to remedy this problem. Among these variants was the so-called *ridgelet transform*. The ridgelet transform uses ideas from wavelet theory and the Radon transform to build a transform that can be oriented to any hyperplane through the origin. Later variants included curvelets and contourlets. These transforms are more localized than the ridgelet transform. Donoho, Candes, and their collaborators have obtaining promising results when applying these transforms to applications in image compression and image de-noising.

Proposed Project

The purpose of this project would be to develop *multi-ridgelets*. We would combine ideas from multiwavelets and ridgelets in the hopes of constructing a transformation that is useful in applications of image processing.

Student assistance in this project is vital. Among the tasks that will be assigned to a group of students are:

> Input data must be preprocessed (change of basis) before application of multiridgelet transform. Students will write the preprocessor module.
>
> In order to understand how the transform works with various types of filters, some symbolic matrix computation routines would be desirable. The students will use Mathematica to develop modules for constructing *generic* transformations from given filters. Using these modules we can learn more about properties obeyed by the input filters.
>
> The students will write the modules that perform the multiridgelet and inverse multiridgelet transform.
>
> The students will write a module that emmulates the JPEG2000 compression standard. In place of the biorthogonal wavelet transform, they will insert their multiridgelet module and determine the effectiveness of the new transform in data compression.

Students will obtain the necessary mathematical background by enrolling in Applied Mathematical Modeling II at the University of St. Thomas. This course has been developed by the PI (support provided by NSF DUE-0442684) and is a semester-long study of discrete wavelet transforms. The computer skills necessary to complete the tasks outlined above will be covered in the Applied Mathematical Modeling I course that is described in this proposal.