

## CSUMS Seminars 11 and 12 - Finite Element Method

Consider the differential equation

$$\frac{d^2u}{dx^2} + \frac{1}{10}(10 - x) = 0 \quad \text{on } 0 < x < 10$$

with boundary conditions

$$\begin{aligned} u &= 10 && \text{at } x = 0 \\ -\frac{du}{dx} &= 0.5 && \text{at } x = 10 \end{aligned}$$

1. Put the differential equation and boundary conditions above into weak form (from seminar 11).
2. Write the element-level unintegrated finite element matrix (from seminar 11).
3. Integrate the matrix from part 2 by hand. In other words, evaluate the integrals explicitly, not using quadrature. You can use *Mathematica* if it is helpful.
4. Discretize the domain  $0 \leq x \leq 10$  into 10 elements (11 nodes) of equal length. This means that you should have an element from 0 to 1, from 1 to 2, etc. Use the finite element method solve the differential equation above. This will involve
  - (a) Setting up a element-level finite element matrix ( $\mathbf{k}^e$ ) and an element-level vector ( $\mathbf{f}^e$ ) for each element. Parts 2 and 3 of this assignment will help you with this.
  - (b) Assembling the matrices and vectors.
  - (c) Solving the resulting system of equations. Solve this system directly, i.e. by inverting the global matrix.
  - (d) Plot the final result.

This assignment is due by noon on Friday, December 12, 2008. Please hand in parts 1, 2, and 3, and email part 4 to Prof. Van Fleet and Prof. Stolarska.