CSUMS Seminar 6: Assignment on conjugate gradient and GMRES methods

Consider the following two linear systems of equations

$$\begin{bmatrix} 2 & -1 & 5 & -3 & 1 & 1 & 4 & 7 & 3 & 5 \\ 3 & -2 & -2 & 1 & 0 & 2 & 0 & 5 & 1 & -4 \\ 7 & 2 & 5 & -2 & -3 & 1 & 5 & 4 & 3 & -1 \\ -1 & -2 & 5 & 3 & 2 & -5 & 0 & 0 & 0 & 1 \\ 5 & 7 & -2 & 0 & 1 & -1 & 0 & 4 & 2 & 0 \\ -1 & -1 & 3 & 6 & 7 & 5 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 3 & 0 & 0 & -2 & 4 & 0 & -1 \\ 4 & 2 & -1 & 4 & 0 & 5 & 0 & -1 & -5 & 1 \\ 0 & 1 & 0 & 1 & 2 & -1 & 5 & 3 & 2 & 1 \\ 6 & -6 & 2 & 1 & 0 & 0 & -1 & 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 142 & 17 & 30 & -1 & -23 & 30 & 35 & 76 & 37 & -19 \\ 17 & 105 & -31 & -9 & -9 & 4 & 15 & 23 & -12 & 13 \\ 30 & -31 & 98 & 7 & 19 & -7 & 41 & 45 & 38 & 27 \\ -1 & -9 & 7 & 86 & 53 & 31 & -24 & -7 & -23 & -14 \\ -23 & -9 & 19 & 53 & 68 & 20 & -1 & 12 & 7 & 12 \\ 30 & 4 & -7 & 31 & 20 & 83 & 4 & 14 & -16 & -5 \\ 35 & 15 & 41 & -24 & -1 & 4 & 71 & 55 & 34 & 24 \\ 76 & 23 & 45 & -7 & 12 & 14 & 55 & 133 & 58 & 9 \\ 37 & -12 & 38 & -23 & 7 & -16 & 34 & 58 & 62 & -1 \\ -19 & 13 & 27 & -14 & 12 & -5 & 24 & 9 & -1 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

You goal in this assignment is to solve one of these systems using Matlab's conjugate gradient algorithm (use the function `pcg`) and GMRES algorithm (use function `gmres`). It is up to you to determine which method is appropriate for which system and to determine how to appropriately formulate the function commands in Matlab. Please use the `help` command to see an explanation of each of the two functions (for example, type `help pcg`).

1. Solve the appropriate system using the conjugate gradient method.
   (a) How many iterations does it take for the system to converge if the default tolerance is used?
   (b) How many iterations does it take for the system to converge if a tolerance of 10^{-2} is used?
   (c) What is the residual norm at the 5^{th} iteration?
2. Solve the appropriate system using GMRES. If you need to input a variable described by 'RESTART', use a value of 10
   (a) How many iterations does it take for the system to converge if the default tolerance is used?
   (b) How many iterations does it take for the system to converge if a tolerance of 10^{-2} is used?
3. What happens when you try to solve the system you think is appropriate for the conjugate gradient method using GMRES? What about when you try to solve the system appropriate for GMRES using conjugate gradient? Why do you think the switch causes the behavior that it does.

Do all of your computations in a Matlab *.m file and email the file to Prof. Stolarska and Prof. Van Fleet by noon on Tuesday 10/28. Please answer the above questions in the email.