Development and Implementation of Adaptive Backward Coupling Metropolis Algorithms (ABCMA) - Arkady Shemyakin

The main goal of the study is to develop new sampling methods of Markov Chain Monte Carlo (MCMC) simulation based on the concepts of backwards coupling and proposal adaptation and apply them to problems of statistical estimation. The objectives are theoretical development and computer implementation of new ABCMA, testing their numerical efficiency and comparing the results to those obtained via previously known methods of estimation.

MCMC algorithms (Gibbs sampling and Metropolis-Hastings sampling) have been extensively used in Bayesian statistical research for the last 30 years. They make it possible to obtain parametric estimates for a wide range of data distributions and prior distributions of parameters. The idea of these algorithms is to start a Markov chain by sampling from a simple "proposal" distribution with transition probabilities that guarantee convergence to the target stationary distribution. If the target is chosen as the posterior distribution of data, the values of the chain give a sample from a distribution close to posterior. Based on this sample, one can obtain the Bayes estimator of the parameters of interest. Obtained estimators may be biased and convergence to the target may be slow due to positive correlations in the chain.

Adaptive algorithms first suggested in the 1990s utilize the idea of gradually modifying the proposal distribution during the run of the chain, accelerating the convergence. Backward coupling approach of Foss and Tweedie [23,24] allows the use of backwards steps to obtain "perfect" sample, eliminating bias. The idea to combine these two approaches to enhance Metropolis algorithm leads to ABCMA, first introduced in Shemyakin (2006).

A group of students will work on the project in following stages:

(a) Study of Bayesian statistics methods and MCMC.

(b) Computer implementation of conventional Metropolis algorithm in Mathematica.

(c) Development of backward coupling methods.

(d) Construction of adaptive proposals.

(e) Construction of ABCMA.

(f) Computer implementation of ABCMA in Mathematica.

(g) Using ABCMA to applied problems of statistical parametric estimation.

The background of undergraduates involved in the project assumes basic knowledge of Calculus and Mathematica. During the work on the project, it is desirable to go through the sequence Math 313/314 (Probability and Mathematical Statistics). Independent study of Bayesian Statistics and MCMC is suggested.

Positive experience in the direction related to the project was evidenced in Summer 2006, when Colin Sullivan and John Engstrom successfully went through stages (a) and (b) - that normally would be a good amount of work for a Summer project. Independent study of Bayesian Statistics and MCMC required approximately one month of work (supported by several introductory lectures). However, the lack of necessary knowledge of Mathematica was probably the biggest hurdle.